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中 三

答案

Properties of Quadrilaterals

Class L

Lam Sir

L 0

1. (a) $AB = AC$

$\therefore \angle ACB = \angle ABC$ (base $\angle$s, isos. $\triangle$)

$= a$

$a + a + 40^o = 180^o$ ($\angle$ sum of $\triangle$)

$2a = 140^o$

$a = \underline{\underline{70^o}}$

(b) $a + 150^o = 180^o$ (adj. $\angle$s on st. line)

$a = \underline{\underline{30^o}}$

$PQ // SR$

$\therefore \angle QPR = \angle SRP$ (alt. $\angle$s, $PQ // SR$)

$b = \underline{\underline{15^o}}$

$PM = PS$

$\therefore \angle PMS = \angle PSM$ (base $\angle$s, isos. $\triangle$)

$= c$

$\angle PMS = a$ (vert. opp. $\angle$s)

$\therefore c = a$

$= \underline{\underline{30^o}}$

2. $a + 6 = 4a$

$3a = 6$

$a = \underline{\underline{2}}$

$c = 4a$

$= \underline{\underline{8}}$

$\frac{b}{2} = 4a$

$b = 8a$

$= \underline{\underline{16}}$

3. (a) $x = \underline{\underline{90^o}}$ (property of rectangle)

$2y + 30^o = 90^o$ (property of rectangle)

$2y = 60^o$

$y = \underline{\underline{30^o}}$

(b) $3x = 90^o$ (property of rectangle)

$x = \underline{\underline{30^o}}$

$x + 4y = 90^o$ (property of rectangle)

$30^o + 4y = 90^o$

$4y = 60^o$

$y = \underline{\underline{15^o}}$

4. (a) $x + 105^\circ = 180^\circ$ (int. $\angle$s, AD // BC)

$x = \underline{\underline{75^\circ}}$

(b) $x = \underline{\underline{128^\circ}}$ (property of //gram)

$7y + 31^\circ + 128^\circ = 180^\circ$ (int. $\angle$s, PS // QR)

$7y = 21^\circ$

$y = \underline{\underline{3^\circ}}$

5. (a) $x + 15^\circ = 90^\circ$ (property of square)

$x = \underline{\underline{75^\circ}}$

$15 - y = 8$

$y = \underline{\underline{7}}$

(b) $2x = 100$

$x = \underline{\underline{50}}$

$(x + 2y)^\circ = 90^\circ$ (property of square)

$50 + 2y = 90$

$2y = 40$

$y = \underline{\underline{20}}$

6.

$AB = CB$	given
$BM = BM$	common sides
$AM = CM$	given
$\therefore\ \underline{\triangle ABM \cong \triangle CBM}$	*SSS*

L 1

1. $\angle ADC = \angle ABC$ (property of //gram)

$= 25^\circ + 35^\circ$

$= \underline{\underline{60^\circ}}$

$\angle BCD + \angle ADC = 180^\circ$ (int. $\angle$s, AD // BC)

$\angle BCE + 40^\circ + 60^\circ = 180^\circ$

$\angle BCE = \underline{\underline{80^\circ}}$

$x - 2 = 9$

$x = \underline{\underline{11}}$

2. $AM = DM$ (property of square)

$\therefore\ 2(x - 1) = 12$

$x - 1 = 6$

$x = \underline{\underline{7}}$

$CM = DM$ (property of square)

$\therefore\ 4y = 12$

$y = \underline{\underline{3}}$

3. $\angle BAD + \angle CDA = 180^o$ (int. $\angle$s, $AB // DC$)

$x + 80^o = 180^o$

$x = \underline{\underline{100^o}}$

$\angle ABC + \angle DCB = 180^o$ (int. $\angle$s, $AB // DC$)

$4y + y + 30^o = 180^o$

$5y = 150^o$

$y = \underline{\underline{30^o}}$

$z = y + 30^o$ (corr. $\angle$s, $AE // BC$)

$= \underline{\underline{60^o}}$

4. (a) $\angle ADC + \angle BCD = 180^o$ (int. $\angle$s, $AD // BC$)

$x + 35^o + 55^o = 180^o$

$x = \underline{\underline{90^o}}$

(b) $\sin \angle BCD = \sin 55^o$

$= \underline{\underline{0.819\ 2}}$ (cor. to 4 sig. fig.)

(c) As $x = 90^o$,

$\therefore$ $\triangle BCD$ is a right-angled triangle.

$\sin \angle BCD = \dfrac{BD}{BC}$

$0.819\ 2 = \dfrac{y}{4}$

$y = \underline{\underline{3.28}}$ (cor. to 3 sig. fig.)

5. $\angle ADC = 90^o$ (property of rectangle)

$\angle CAD + \angle ADC + \angle ACD = 180^o$ ($\angle$ sum of $\triangle$)

$4x + 90^o + 2x = 180^o$

$6x = 90^o$

$x = \underline{\underline{15^o}}$

$\angle ABC = 90^o$ (property of rectangle)

$BC = AD$

$= y$

$AB^2 + BC^2 = AC^2$ (Pyth. theorem)

$10^2 + y^2 = (2y)^2$

$100 = 3y^2$

$y^2 = \dfrac{100}{3}$

$y = \sqrt{\dfrac{100}{3}}$

$= \underline{\underline{5.77}}$ (cor. to 3 sig. fig.)

6. $AD = AB$

$s = \underline{\underline{6}}$

$u = \underline{\underline{90^\circ}}$ (property of rhombus)

$\angle BAM + \angle ABM + \angle AMB = 180^\circ$ ($\angle$ sum of $\triangle$)

$t + 2t + 90^\circ = 180^\circ$

$3t = 90^\circ$

$t = \underline{\underline{30^\circ}}$

$\angle DAM = \angle BAM$ (property of rhombus)

$w = t$

$= \underline{\underline{30^\circ}}$

$\angle CBM = \angle ABM$ (property of rhombus)

$v = 2t$

$= \underline{\underline{60^\circ}}$

7.

PQRS is a parallelogram.	given
$MQ = MS$	property of //gram
$PM = RM$	property of //gram
$PQ = RS$	property of //gram
$\therefore$ $\underline{\triangle PMQ \cong \triangle RMS}$	*SSS*

8.

$AB = DC$	property of //gram
$BE = DF$	given
$\therefore AE = AB - BE$	
$= DC - DF$	
$= FC$	
$AB \parallel DC$	property of //gram
$\therefore AE \parallel FC$	
$\therefore$ <u>*AECF* is a parallelogram.</u>	2 sides equal and //

A 1

1. (a) Given that $AB = AD$ and $BC = DC$,

$$\therefore \begin{cases} x = 4 \\ x + 3 = y \end{cases}$$

Substitute $x = 4$ into $x + 3 = y$,

$4 + 3 = y$

$y = 7$

$\therefore \quad x = \underline{\underline{4}}, \ y = \underline{\underline{7}}$

(b) Perimeter of kite *ABCD*

$= AB + BC + CD + DA$

$= 4 + 7 + 7 + 4$

$= \underline{\underline{22}}$

2. (a) Given that $AB = AD$ and $BC = DC$,

$$\begin{cases} x - y = 5 \text{..................}(1) \\ x + 4 = 2(y+1) \text{........}(2) \end{cases}$$

From (2),

$x + 4 = 2y + 2$

$x - 2y = -2$ ....(3)

(1) − (3), $y = \underline{\underline{7}}$

Substitute $y = 7$ into (1),

$x - 7 = 5$

$x = \underline{\underline{12}}$

(b) $BC = x + 4$

$= 12 + 4$

$= 16$

Perimeter of kite *ABCD*

$= AB + BC + CD + DA$

$= 5 + 16 + 16 + 5$

$= \underline{\underline{42}}$

3. Given that AB // DC,

$120° - y + 4y = 180°$ (prop. of trapezium)

$3y = 60°$

$y = \underline{\underline{20°}}$

$x + y + 5° = 180°$ (prop. of trapezium)

$x + 20° + 5° = 180°$

$x = \underline{\underline{155°}}$

4. $AD = BC$ (opp. sides of //gram)

$13 = y + 3$

$y = \underline{\underline{10}}$

$AB = DC$ (opp. sides of //gram)

$x + y = 30$

$x + 10 = 30$

$x = \underline{\underline{20}}$

5. $AD = BC$ (opp. sides of //gram)

$6 + x = 2 - x$

$2x = -4$

$x = \underline{\underline{-2}}$

$AB = DC$ (opp. sides of //gram)

$9 + y = 2(y - 3)$

$9 + y = 2y - 6$

$y = \underline{\underline{15}}$

6. $\because$ $AB = DC$ and $AD = BC$ (opp. sides of //gram)

$\therefore \begin{cases} x = y + 1 \text{..................(1)} \\ 5 - x = y \text{..................(2)} \end{cases}$

(1) + (2),

$5 = 2y + 1$

$2y = 4$

$y = \underline{\underline{2}}$

Substitute $y = 2$ into (1),

$x = 2 + 1$

$= \underline{\underline{3}}$

7. $AO = OC$ (diag. of //gram)

$4x = 15 - x$

$5x = 15$

$x = \underline{\underline{3}}$

$BO = OD$ (diag. of //gram)

$y + 4 = 12$

$y = \underline{\underline{8}}$

8. $x + x + 80° = 180°$ (int. $\angle$s, AD // BC)

$2x = 100°$

$x = \underline{\underline{50°}}$

$y + x = 180°$ (int. $\angle$s, DC // AB)

$y + 50° = 180°$

$y = \underline{\underline{130°}}$

9. $\angle DCB = 114°$ (corr. $\angle$s, BE // CD)

$\angle DCE + 60° = 114°$

$\angle DCE = 54°$

In ΔOCD,

$x = \angle DCO + 32°$ (ext. $\angle$ of Δ)

$= 54° + 32°$

$= \underline{\underline{86°}}$

$\angle DEC = 60°$ (alt. $\angle$s, ED // BC)

In ΔODE,

$x + y + \angle DEO = 180°$ ($\angle$ sum of Δ)

$86° + y + 60° = 180°$

$y = \underline{\underline{34°}}$

10. $2y + (3y - 80°) = 180°$ (int. $\angle$s, AB // DC)

$5y - 80° = 180°$

$5y = 260°$

$y = \underline{\underline{52°}}$

$x = 3y - 80°$ (opp. $\angle$s of //gram)

$= 3(52°) - 80°$

$= \underline{\underline{76°}}$

11. $AB = DC$ (prop. of rectangle)

$12 = 3x$

$x = \underline{\underline{4}}$

$AD = BC$ (prop. of rectangle)

$5 = x - y$

$4 - y = 5$

$y = \underline{\underline{-1}}$

In ΔABD,

$BD^2 = 12^2 + 5^2$ (Pyth. theorem)

$BD^2 = 169$

$BD = 13$

$\because$ $DE = EB$ (prop. of rectangle)

$\therefore$ $2z = 13$

$z = \underline{\underline{6.5}}$

12. $\angle BAD = 90°$ (definition of rectangle)

$x + 32° = 90°$

$x = \underline{\underline{58°}}$

$\because$ $AC = BD$ (prop. of rectangle)

E is the mid-point of AC and BD. (prop. of rectangle)

$\therefore$ $AE = DE$

$\therefore$ $y = x$ (base $\angle$s, isos. Δ)

$= \underline{\underline{58°}}$

In ΔADE,

$\angle AED + x + y = 180°$ ($\angle$ sum of Δ)

$\angle AED + 58° + 58° = 180°$

$\angle AED = 64°$

$\therefore$ $z = \angle AED$ (vert. opp. $\angle$s)

$= \underline{\underline{64°}}$

13. $x = \underline{\underline{90°}}$ (prop. of rhombus)

In ΔAED,

$z + 60° = x$ (ext. $\angle$ of Δ)

$z + 60° = 90°$

$z = \underline{\underline{30°}}$

$\because$ $BC \,//\, AD$ (definition of rhombus)

$\therefore$ $y = z$ (alt. $\angle$s, $BC \,//\, AD$)

$= \underline{\underline{30°}}$

14. $\because$ $AD = DC$ (definition of rhombus)

$\therefore$ $\angle DAC = \angle DCA$ (base $\angle$s, isos. Δ)

$= 24°$

$\angle BTA = 90°$ (given)

In ΔAST,

$\angle BSA = \angle TAS + \angle STA$ (ext. $\angle$ of Δ)

$= 24° + 90°$

$= \underline{\underline{114°}}$

15. $OA = OC$ (prop. of rhombus)

$= \dfrac{AC}{2}$

$= \dfrac{24\text{ cm}}{2}$

$= 12\text{ cm}$

$OB = OD$ (prop. of rhombus)

$= \dfrac{BD}{2}$

$= \dfrac{10\text{ cm}}{2}$

$= 5\text{ cm}$

$\angle AOD = 90°$ (prop. of rhombus)

In ΔOAD,

$AD^2 = OA^2 + OD^2$ (Pyth. theorem)

$= (12^2 + 5^2)\text{ cm}^2$

$= 169\text{ cm}^2$

$AD = 13\text{ cm}$

$\therefore$ Perimeter of the rhombus $= (4 \times 13)\text{ cm}$

$= \underline{\underline{52\text{ cm}}}$

16. **(a)** $\because$ AB // DC (definition of rectangle)

$\therefore$ $15x = 9x + 18°$ (alt. $\angle$s, AB // DC)

$6x = 18°$

$x = \underline{\underline{3°}}$

(b) $\therefore$ $\angle BAC = 15(3°)$

$= 45°$

$\angle ACD = 9(3°) + 18°$

$= 45°$

$\because$ $\angle BCD = 90°$ (definition of rectangle)

$\therefore$ $\angle BCA + \angle ACD = 90°$

$\angle BCA + 45° = 90°$

$\angle BCA = 45°$

$\because \quad \angle BAC = \angle BCA = 45°$

$\therefore \quad AB = BC$ (sides opp. eq. ∠s)

$\because \quad AB = DC$ and $BC = AD$ (prop. of rectangle)

$\therefore \quad AB = BC = DC = AD$

$\therefore \quad ABCD$ is a square.

17. (a) $\because \quad AB = AD$

$\therefore \quad 3a = 12$

$a = \underline{\underline{4}}$

(b) $BC = (6a + 2)$ cm

$= [6(4) + 2]$ cm

$= 26$ cm

$DC = (5a + 6)$ cm

$= [5(4) + 6]$ cm

$= 26$ cm

$\because \quad AB = AD$ and $BC = DC$

$\therefore \quad ABCD$ is a kite.

18. $DE \mathbin{/\mkern-5mu/} BC$ (mid-point theorem)

$\therefore \quad x = \underline{\underline{18°}}$ (corr. ∠s, $DE \mathbin{/\mkern-5mu/} BC$)

$DE = \frac{1}{2} BC$ (mid-point theorem)

$y = \frac{1}{2} \times 10$

$= \underline{\underline{5}}$

19. $DE \mathbin{/\mkern-5mu/} BC$ (mid-point theorem)

$\therefore \quad \angle ACB = x$ (corr. ∠s, $DE \mathbin{/\mkern-5mu/} BC$)

In ΔABC,

$78° + 56° + x = 180°$ (∠ sum of Δ)

$\therefore \quad x = \underline{\underline{46°}}$

$DE = \frac{1}{2} BC$ (mid-point theorem)

$5 = \frac{1}{2} y$

$\therefore \quad y = \underline{\underline{10}}$

20. $\dfrac{AC}{CE} = \dfrac{BD}{DF}$ (intercept theorem)

$\dfrac{1\text{ cm}}{2\text{ cm}} = \dfrac{x\text{ cm}}{3\text{ cm}}$

$\therefore \quad x = \underline{\underline{1.5}}$

$\dfrac{CE}{EG} = \dfrac{DF}{FH}$ (intercept theorem)

$\dfrac{2\text{ cm}}{3\text{ cm}} = \dfrac{3\text{ cm}}{y\text{ cm}}$

$\therefore \quad y = \underline{\underline{4.5}}$

21. $\because$ $AS = SR$ and $AP = PQ$

$\therefore \quad PS = \dfrac{1}{2}QR$ (mid-point theorem)

$x\text{ cm} = \dfrac{1}{2} \times 3\text{ cm}$

$x = \underline{\underline{1.5}}$

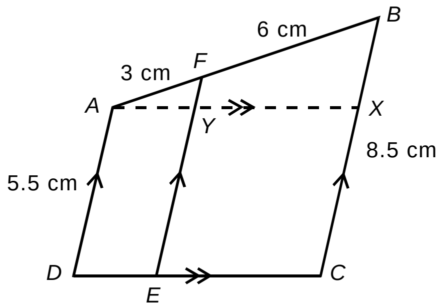
$\because$ $AR = RC = 6$ cm and $AQ = QB = 8$ cm

$\therefore \quad QR = \dfrac{1}{2}BC$ (mid-point theorem)

$3\text{ cm} = \dfrac{1}{2} \times y\text{ cm}$

$y = \underline{\underline{6}}$

22. Draw the straight line AX such that AX // DC and it meets EF at Y.



$XC = YE = AD = 5.5$ cm (opp. sides of //gram)

$BX = 8.5$ cm $-$ 5.5 cm

$= 3$ cm

In ΔABX and ΔAFY,

$\angle BAX = \angle FAY$ (common angle)

$\angle AXB = \angle AYF$ (corr. $\angle$s, EF // CB)

$\angle ABX = \angle AFY$ (corr. $\angle$s, EF // CB)

$\therefore \quad \Delta ABX \sim \Delta AFY$ (equiangular)

$\therefore \quad \dfrac{FY}{BX} = \dfrac{AF}{AB}$ (corr. sides, ~ Δs)

$\dfrac{FY}{3\text{ cm}} = \dfrac{3\text{ cm}}{(3+6)\text{ cm}}$

$FY = 1\text{ cm}$

$\therefore \quad EF = FY + YE$

$= 1\text{ cm} + 5.5\text{ cm}$

$= \underline{\underline{6.5\text{ cm}}}$

L 2

1. (a) $BM = AM$ (property of rectangle)

$\therefore \angle ABM = \angle BAM$ (base $\angle$s, isos. $\triangle$)

$= 54^\circ$

$\angle BMD = \angle BAM + \angle ABM$ (ext. $\angle$ of $\triangle$)

$= 54^\circ + 54^\circ$

$= \underline{\underline{108^\circ}}$

(b) $\angle BMC = 60^\circ$ (property of equil. $\triangle$)

$\therefore \angle CMD = 108^\circ - 60^\circ$

$= 48^\circ$

$DM = BM$ (property of rectangle)

$= CM$

$\therefore \angle CDM = \angle DCM$ (base $\angle$s, isos. $\triangle$)

$\angle CDM + \angle DCM + \angle CMD = 180^\circ$ ($\angle$ sum of $\triangle$)

$2\angle CDM + 48^\circ = 180^\circ$

$\angle CDM = \underline{\underline{66^\circ}}$

2. (a) $\angle BAE = \angle ADC$ (corr. $\angle$s, AB // DC)

$= 68^\circ$

$BE = BA$ (given)

$\therefore \angle BEA = \angle BAE$ (base $\angle$s, isos. $\triangle$)

$= 68^\circ$

$\angle ABE + \angle BAE + \angle BEA = 180^\circ$ ($\angle$ sum of $\triangle$)

$\angle ABE + 68^\circ + 68^\circ = 180^\circ$

$\angle ABE = \underline{\underline{44^\circ}}$

(b) $\angle ABC = \angle ADC$ (property of rhombus)

$= 68^\circ$

$\angle CBE = 68^\circ + 44^\circ$

$= 112^\circ$

$\because BC = BA$

$\therefore \quad = BE$ (given)

$\therefore \angle BEC = \angle BCE$ (base $\angle$s, isos. $\triangle$)

$\angle BCE + \angle BEC + \angle CBE = 180^o$ ($\angle$ sum of $\triangle$)

$2\angle BCE + 112^o = 180^o$

$\angle BCE = \underline{\underline{34^o}}$

3. (a)

$EB = FD$	given
$\angle CBE = \angle CDF = 90^o$	property of square
$BC = DC$	
$\therefore \underline{\triangle BCE \cong \triangle DCF}$	*SAS*

(b) $\angle BCE = \angle DCF$ (corr. $\angle$s, $\triangle BCE \cong \triangle DCF$)

$\angle BCD = 90^o$ (property of square)

$\angle BCE + \angle DCF + 64^o = 90^o$

$2\angle BCE = 26^o$

$\angle BCE = 13^o$

$\angle BEC + \angle CBE + \angle BCE = 180^o$ ($\angle$ sum of $\triangle$)

$\angle BEC + 90^o + 13^o = 180^o$

$\angle BEC = \underline{\underline{77^o}}$

4. (a) $\angle BAD = \angle BCD$ (property of //gram)

$= y$

$\angle BPD = \angle ADP + \angle DAP$ (ext. $\angle$ of $\triangle$)

$\underline{z = x + y}$

(b) $\angle PDC = \angle ADP$ (given)

$= 58^o$

$\angle ADC + \angle BCD = 180^o$ (int. $\angle$s, AD // BC)

$58^o + 58^o + y = 180^o$

$y = \underline{\underline{64^o}}$

$z = x + y$ (proved)

$= 58^o + 64^o$

$= \underline{\underline{122^o}}$

5. (a)

$\angle APB = \angle CQD = 90^o$	given
$\angle BAP = \angle DCQ$	alt. $\angle$s, AB // DC
$AB = CD$	property of //gram
$\therefore \underline{\triangle APB \cong \triangle CQD}$	*AAS*

(b)

$PB = DQ$	corr. sides, $\cong \triangle$s
$\angle BPQ = \angle DQP = 90^o$	given
$\therefore BP$ // DQ	alt. $\angle$s equal
$\therefore$ <u>*PBQD* is a parallelogram.</u>	2 sides equal and //

6. $PS = QR$ — property of //gram

$PD + DS = QB + BR$

$PD + DS = DS + BR$

$PD = BR$

In $\triangle APD$ and $\triangle CRB$,

$AP = CR$ — given

$\angle APD = \angle CRB$ — property of //gram

$PD = BR$ — proved

$\therefore \triangle APD \cong \triangle CRB$

$\therefore AD = CB$ — corr. sides, $\cong \triangle$s

$PQ = SR$ — property of //gram

$PA + AQ = SC + CR$

$PA + AQ = SC + PA$

$AQ = SC$

In $\triangle AQB$ and $\triangle CSD$,

$QB = SD$ — given

$\angle AQB = \angle CSD$ — property of //gram

$AQ = CS$ — proved

$\therefore \triangle AQB \cong \triangle CSD$

$\therefore AB = CD$ — corr. sides, $\cong \triangle$s

$\therefore$ <u>$ABCD$ is a parallelogram.</u> — opp. sides equal

7. (a) $MN \parallel BC$ (mid-pt. theorem)

$\therefore x = 55^\circ$

(b) $BF \parallel CG \parallel DH$ (given)

$BC = CD$ (given)

$\therefore FG = GH$ (intercept theorem)

$x = \underline{\underline{7}}$

$AE \parallel BF \parallel CG$ (given)

$EF = FG$ (proved)

$\therefore AB = BC$ (intercept theorem)

$AB = 5$

$y = AB + BC + CD$

$= 5 + 5 + 5$

$= \underline{\underline{15}}$

(c) $BE \parallel CD$ (given)

$AB = BC$ (given)

$\therefore AE = ED$ (intercept theorem)

$x = y$

$x + y = 28$

$2x = 28$

$x = \underline{\underline{14}}$

$y = x$

$= \underline{\underline{14}}$

8. $BP = AP$	given
$BQ = CQ$	given
$\therefore$ $PQ // AC$	mid-pt. theorem
i.e. $PQ // AR$	
$CR = AR$	given
$CQ = BQ$	given
$\therefore$ $RQ // AB$	mid-pt. theorem
i.e. $RQ // AP$	
$PQ // AR$ and $RQ // AP$	
$\therefore$ $\underline{APQR \text{ is a parallelogram.}}$	

A 2

23. **(a)** Area of $\Delta ABC = \frac{1}{2} \times AC \times BE$

$= [\frac{1}{2} \times (7 + 10) \times 24] \text{ cm}^2$

$= 204 \text{ cm}^2$

$\therefore$ Area of kite $ABCD = (204 \times 2) \text{ cm}^2$

$= \underline{\underline{408 \text{ cm}^2}}$

(b) In ΔABE,

$AB^2 = AE^2 + BE^2$ (Pyth. theorem)

$= (7^2 + 24^2) \text{ cm}^2$

$AB = 25 \text{ cm}$

In ΔBCE,

$BC^2 = BE^2 + CE^2$ (Pyth. theorem)

$= (24^2 + 10^2) \text{ cm}^2$

$BC = 26 \text{ cm}$

$\because$ $AB = AD = 25$ cm and $BC = DC = 26$ cm (definition of kite)

$\therefore$ Perimeter of kite $ABCD = [2 \times (AB + BC)] \text{ cm}$

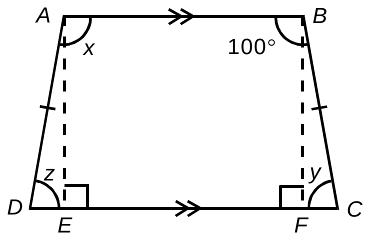
$= [2 \times (25 + 26)] \text{ cm}$

$= \underline{\underline{102 \text{ cm}}}$

24. $100^\circ + y = 180^\circ$ (prop. of trapezium)

$\therefore \quad y = \underline{\underline{80^\circ}}$

Draw two perpendicular lines from points A and B to DC such that they intersect DC at E and F respectively.



In ΔADE and ΔBCF,

$AE = BF$ (height of trapezium)

$\angle AED = \angle BFC = 90^\circ$ (by construction)

$AD = BC$ (given)

$\therefore \quad \Delta ADE \cong \Delta BCF$ (R.H.S.)

$\angle ADE = \angle BCF$ (corr. $\angle$s, $\cong$ Δs)

i.e. $z = y$

$= \underline{\underline{80^\circ}}$

$x + z = 180^\circ$ (prop. of trapezium)

$x + 80^\circ = 180^\circ$

$\therefore \quad x = \underline{\underline{100^\circ}}$

25. $AB = BC$ (definition of square)

In ΔABC,

$AB^2 + BC^2 = 6^2 \text{ cm}^2$ (Pyth. theorem)

$AB^2 + AB^2 = 36 \text{ cm}^2$

$2AB^2 = 36 \text{ cm}^2$

$AB^2 = 18 \text{ cm}^2$

$\therefore$ Area of square $ABCD = AB^2$

$= \underline{\underline{18 \text{ cm}^2}}$

26. $\because \quad BE = DE$

$\therefore \quad \angle EBD = \angle EDB$ (base $\angle$s, isos.)

In ΔBDE,

$$\angle DBE + \angle EDB + 40^\circ = 180^\circ \quad (\angle \text{ sum of } \Delta)$$
$$2\angle DBE = 140^\circ$$
$$\angle DBE = \underline{\underline{70^\circ}}$$

$\because$ $\angle CBD = 45^\circ$ (prop. of square)

$\therefore$ $\angle CBE = \angle DBE - \angle CBD$
$= 70^\circ - 45^\circ$
$= \underline{\underline{25^\circ}}$

27. **(a)** $\angle ADE = \angle EDC = x$ (given)

$\angle BCE = \angle ECD = y$ (given)

$\because$ $AD \,//\, BC$ (definition of //gram)

$\therefore$ $\angle ADC + \angle BCD = 180^\circ$ (int. $\angle$s, $AD \,//\, BC$)

$2x + 2y = 180^\circ$

$x + y = 90^\circ$

(b) In ΔEDC,

$\angle DEC + x + y = 180^\circ$ ($\angle$ sum of Δ)

$\angle DEC + 90^\circ = 180^\circ$ [By the result of **(a)**]

$\therefore$ $\angle DEC = \underline{\underline{90^\circ}}$

28. $AD = CD$ (definition of square)

$\angle EAD = \angle BCD = 90^\circ$ (definition of square)

$\because$ $\angle BCD + \angle FCD = 180^\circ$ (adj. $\angle$s on st. line)
$90^\circ + \angle FCD = 180^\circ$
$\angle FCD = 90^\circ$

$\therefore$ $\angle EAD = \angle FCD$

Let $\angle ADE = x$.

$\because$ $\angle ADC = 90^\circ$ (definition of square)

$\therefore$ $\angle EDC + x = 90^\circ$

$\angle EDC = 90^\circ - x$

$\because$ $\angle EDF = 90^\circ$ (given)

$\therefore$ $\angle CDF + \angle EDC = 90^\circ$

$\angle CDF = 90^\circ - \angle EDC$

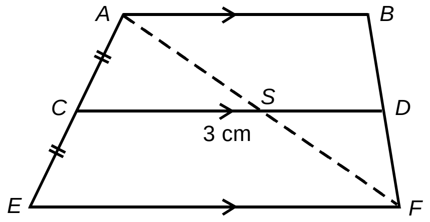
$= 90^\circ - (90^\circ - x)$

$= x$

$\therefore$ $\angle ADE = \angle CDF$

$\therefore$ $\Delta ADE \cong \Delta CDF$ (A.S.A.)

29. Join AF to cut CD at S.



$AB \,//\, CD \,//\, EF$ (given)

$AC = CE$ (given)

$\therefore$ $AS = SF$ and $BD = DF$ (intercept theorem)

In ΔAEF, $CS = \frac{1}{2}EF$ (mid-point theorem)

$EF = 2CS$

In ΔABF, $SD = \frac{1}{2}AB$ (mid-point theorem)

$AB = 2SD$

$\therefore$ $AB + EF = 2SD + 2CS$

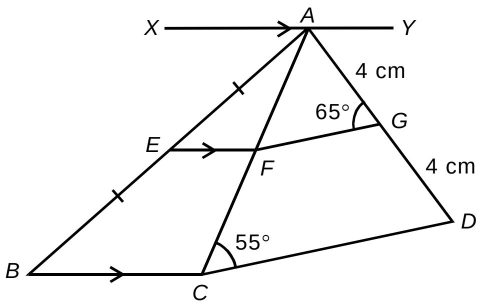
$= 2(SD + CS)$

$= 2CD$

$= 2 \times 3\text{ cm}$

$= \underline{\underline{6\text{ cm}}}$

30. **(a)** Draw the straight line XY passing through A and parallel to BC.



In ΔABC,

$AE = EB$ (given)

$\therefore$ $AF = FC$ (intercept theorem)

In ΔACD,

$AG = GD = 4$ cm (given)

$AF = FC$ (proved)

$\therefore$ $FG \,//\, CD$ (mid-point theorem)

(b) $\angle ADC = \angle AGF$ (corr. $\angle$s, $FG \,//\, CD$)

$= 65°$

In ΔACD,

$\angle CAD + 55° + 65° = 180°$ ($\angle$ sum of Δ)

$\angle CAD = \underline{\underline{60°}}$

L 3

1. $\angle ABF = 45^{o}$ (property of square)

$\angle BAG + \angle ABG = \angle BGD$ (ext. $\angle$ of $\triangle$)

$\angle BAG + 45^{o} = 83^{o}$

$\angle BAG = 38^{o}$

$\angle BCD = \angle BAG$ (property of parallelogram)

$= \underline{\underline{38^{o}}}$

2. $\angle PSR + \angle QRS = 180^{o}$ (int. $\angle$s, PS // QR)

$76^{o} + \angle QRS = 180^{o}$

$\angle QRS = 104^{o}$

$\angle PRS = \angle PRQ$ (property of rhombus)

$\angle PRS + \angle PRQ = \angle QRS$

$2\angle PRS = 104^{o}$

$\angle PRS = 52^{o}$

$\angle RTV = \angle PTQ$ (vert. opp. $\angle$s)

$= 84^{o}$

$\angle RVT + \angle TRV + \angle RTV = 180^{o}$ ($\angle$ sum of $\triangle$)

$\angle RVT + 52^{o} + 84^{o} = 180^{o}$

$\angle RVT = 44^{o}$

i.e. $\angle RVQ = \underline{\underline{44^{o}}}$

3. $\angle ADC = 90^{o}$ (property of rectangle)

$AC^2 = AD^2 + CD^2$ (Pyth. theorem)

$AC^2 = 3^2 + (\sqrt{7})^2$

$AC^2 = 16$

$AC = 4$

$AE = BE = CE = DE$ (property of rectangle)

$\therefore BE = \frac{1}{2}AC$

$= \frac{1}{2}(4)$

$= \underline{\underline{2}}$

$\tan \angle CAD = \frac{\sqrt{7}}{3}$

$\angle CAD = 41.410^{o}$ (cor. to 5 sig. fig.)

$\angle EDA = \angle EAD$	(base $\angle$s, isos. $\triangle$)
$= 41.410^\circ$	
$\angle CED = \angle EAD + \angle EDA$	(ext. $\angle$ of $\triangle$)
$x = 41.410^\circ + 41.410^\circ$	
$= \underline{\underline{82.8^\circ}}$ (cor. to 3 sig. fig.)	

4.

$BA = BC$	given
$BP + PA = BQ + QC$	
$BP + PA = BP + QC$	
$\therefore PA = QC$	
$BA = BC$	given
$\therefore \angle BAC = \angle ACB$	base $\angle$s, isos. $\triangle$
$\angle ARP = \angle ACB$	corr. $\angle$s, PQ // BC
$\therefore \angle ARP = \angle BAC$	
$\therefore PA = PR$	sides opp. equal $\angle$s
$\therefore QC = PR$	
QC // PR	given
$\therefore$ <u>*PQCR* is a parallelogram.</u>	2 sides equal and //

5. (a) In $\triangle RSY$ and $\triangle RMY$,

$RY = RY$	common side
$\angle RMY = 90^\circ$	given
$\angle RSY = 90^\circ$	property of square
$\therefore \angle RSY = \angle RMY$	
$RS = RM$	given
$\therefore \underline{\triangle RSY \cong \triangle RMY}$	*RHS*

(b)

$\angle SRP = 45^\circ$	(property of square)
$\angle SRY = \angle MRY$	(corr. $\angle$s, $\cong\triangle$s)

$\therefore \angle MRY = \frac{1}{2}\angle SRP$

$= 22.5^\circ$

$QR = RS$

$= RM$

Similarly, $\angle MRX = 22.5^\circ$.

$\angle XRY = \angle MRX + \angle MRY$

$= 22.5^\circ + 22.5^\circ$

$= \underline{\underline{45^\circ}}$

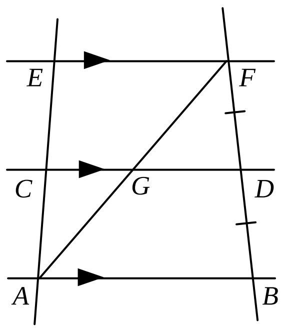
6. (a)

$\angle BAE = \angle AHE + \angle AEH$	ext. $\angle$ of $\triangle$
$= 30^{o} + 30^{o}$	
$= 60^{o}$	
$AE = BE = CE = DE$	property of rectangle
$\angle ABE = \angle BAE$	base $\angle$s, isos. $\triangle$
$= 60^{o}$	
$\angle ABE + \angle BAE + \angle AEB = 180^{o}$	$\angle$ sum of $\triangle$
$60^{o} + 60^{o} + \angle AEB = 180^{o}$	
$\angle AEB = 60^{o}$	
$\angle FEC = \angle AEB$	vert. opp. $\angle$s
$= 60^{o}$	
$\angle DCE = \angle BAE$	alt $\angle$s, AB // DC
$= 60^{o}$	
In $\triangle HEB$ and $\triangle FCE$,	
$\angle HBE = \angle FEC = 60^{o}$	proved
$\angle HEB = \angle AEH + \angle AEB$	
$= 30^{o} + 60^{o} = 90^{o}$	
$\angle FCE = \angle FCD + \angle ECD$	
$= 30^{o} + 60^{o} = 90^{o}$	
$\therefore \angle HEB = \angle FCE$	
$BE = EC$	property of rectangle
$\therefore$ <u>$\triangle CEF \cong \triangle EBH$</u>	*ASA*

(b)

$EF = BH$	corr. sides, $\cong \triangle$s
$= GH$	
$EH = CF$	corr. sides, $\cong \triangle$s
$= GF$	
$\therefore$ $EFGH$ is a parallelogram.	opp. sides equal
$\angle HEB = 90^{o}$	
$\therefore \angle HEF = 90^{o}$	adj. $\angle$s on st. line
$\therefore$ <u>$EFGH$ is a rectangle.</u>	

7. Join AF. Suppose AF and CD meet at G.



AB // CD // EF and $FD = DB$	given
$\therefore FG = AG$	intercept theorem
and $EC = AC$	intercept theorem
$\therefore DG = \frac{1}{2}AB$	mid-pt. theorem
and $CG = \frac{1}{2}EF$	mid-pt. theorem

$CG + DG = CD$

$\frac{1}{2}EF + \frac{1}{2}AB = CD$

$\frac{1}{2}(AB + EF) = CD$

$\underline{AB + EF = 2CD}$

8. (a) In $\triangle ACD$,

$AP = DP$	given
$PR // DC$	given
$\therefore\ AR = RC$	intercept theorem
$BS = CS$	given
$AB // RS$	
i.e. $AB // DC$.	mid-pt. theorem

(b)

$DP = AP$	(given)
$PQ // AB$	(proved)
$\therefore\ DQ = BQ$	(intercept theorem)
In $\triangle ATB$ and $\triangle CTD$,	
$\angle BAT = \angle DCT$	(alt. $\angle$s, $AB // DC$)
$\angle ABT = \angle CDT$	(alt. $\angle$s, $AB // DC$)
$\angle ATB = \angle CTD$	(vert. opp. $\angle$s)
$\therefore\ \triangle ATB \sim \triangle CTD$	(AAA)
$\therefore\ \frac{AT}{CT} = \frac{BT}{DT}$	(corr. sides, $\sim\triangle$s)

$\frac{18}{45} = \frac{BT}{40}$

$BT = 16$

$BQ = BT + QT$

$\therefore\ DQ = 16 + QT$

$DQ + QT = DT$

$16 + QT + QT = 40$

$2QT = 24$

$QT = \underline{\underline{12}}$

B

31. (a) $\because$ ADE is an equilateral triangle.

$\therefore$ $\angle EDA = 60°$

$\because$ $\angle ADC = 90°$ (definition of square)

$\therefore$ $\angle EDC = \angle EDA + \angle ADC$

$= 60° + 90°$

$= \underline{\underline{150°}}$

(b) $\because$ ADE is an equilateral triangle.

$\therefore$ $DE = DA$

$\because$ $DC = DA$ (definition of square)

$\therefore$ $DE = DC$

$\therefore$ $\angle DEC = \angle DCE$ (base $\angle$s, isos. Δ)

In ΔDEC,

$\angle DEC + \angle DCE + \angle EDC = 180°$ ($\angle$ sum of Δ)

$$2\angle DCE + 150° = 180° \quad \text{[By the result of } \textbf{(a)}\text{]}$$
$$2\angle DCE = 30°$$
$$\angle DCE = 15°$$

$\because$ $\angle DCA = 45°$ (prop. of square)

$\therefore$ $\angle ECA = \angle DCA - \angle DCE$

$$= 45° - 15°$$
$$= \underline{\underline{30°}}$$

32. (a) In ΔADX and ΔCBY,

$\because$ $AD \mathbin{//} BC$ (definition of //gram)

$\therefore$ $\angle DAX = \angle BCY$ (alt. $\angle$s, AD // BC)

$AX = CY$ (given)

$AD = CB$ (opp. sides of //gram)

$\therefore$ $\Delta ADX \cong \Delta CBY$ (S.A.S.)

(b) $\because$ $\Delta ADX \cong \Delta CBY$ (proved)

$\therefore$ $DX = BY$ (corr. sides, $\cong$ Δs)

$\because$ $AB \mathbin{//} DC$ (definition of //gram)

$\therefore$ $\angle BAX = \angle DCY$ (alt. $\angle$s, AB // DC)

$AX = CY$ (given)

$AB = CD$ (opp. sides of //gram)

$\therefore$ $\Delta ABX \cong \Delta CDY$ (S.A.S.)

$\therefore$ $BX = DY$ (corr. sides, $\cong$ Δs)

$\therefore$ $BXDY$ is a parallelogram. (opp. sides eq.)

Alternative method:

$\angle ADO = \angle CBO$ (alt. $\angle$s, AD // BC)

$\because$ $\Delta ADX \cong \Delta CBY$ (proved)

$\therefore$ $DX = BY$ (corr. sides, $\cong$ Δs)

$\angle ADX = \angle CBY$ (corr. $\angle$s, $\cong$ Δs)

$\because$ $\angle XDO = \angle ADO - \angle ADX$

$$= \angle CBO - \angle CBY$$

$= \angle YBO$

$\therefore$ $XD \mathbin{/\mkern-3mu/} BY$ (alt. $\angle$s, eq.)

$\therefore$ $BXDY$ is a parallelogram. (opp. sides eq. and //)

MC 1

1. The answer is D.

$\angle BCD + \angle ADC = 180^o$ (int. $\angle$s, AD // BC)

$x + 90^o = 180^o$

$x = \underline{\underline{90^o}}$

$\angle ABC + \angle BAD = 180^o$ (int. $\angle$s, AD // BC)

$y + x - 35^o = 180^o$

$y + 90^o - 35^o = 180^o$

$y = \underline{\underline{125^o}}$

2. The answer is A.

$6a = 90^o$ (property of square)

$a = \underline{\underline{15^o}}$

$a + b = 90^o$ (property of square)

$15^o + b = 90^o$

$b = \underline{\underline{75^o}}$

3. The answer is C.

$BC = BD$ (given)

$\therefore$ $\angle BDC = \angle BCD$ (base $\angle$s, isos. $\triangle$)

$= 75^o$

$\angle ADC + \angle BCD = 180^o$ (int. $\angle$s, AD // BC)

$\angle ADB + 75^o + 75^o = 180^o$

$\angle ADB = \underline{\underline{30^o}}$

4. The answer is B.

$\angle LMN = 90^o$ (property of rectangle)

$MN^2 + LM^2 = LN^2$

$x^2 + (2x)^2 = 20^2$

$5x^2 = 400$

$x^2 = 80$

$x = \underline{\underline{8.94}}$ (cor. to 3 sig. fig.)

5. The answer is C.

6. The answer is C.

7. The answer is D.

In $\triangle AFD$ and $\triangle CFE$,

$\angle AFD = \angle CFE$ (vert. opp. $\angle$s)

$\angle DAF = \angle ECF$ (alt. $\angle$s, $AD // BC$)

$\angle ADF = \angle CEF$ (alt. $\angle$s, $AD // BC$)

$\therefore \triangle AFD \sim \triangle CFE$ (AAA)

$\therefore \frac{AF}{CF} = \frac{DF}{EF}$ (corr. sides, ~$\triangle$s)

$\frac{8}{6} = \frac{4}{EF}$

$EF = 3$

$\therefore DE = DF + EF$

$= 4 + 3$

$= \underline{\underline{7}}$

8. The answer is B.

$\angle QPS + \angle PSR = 180^o$ (int. $\angle$s, $PQ // SR$)

$105^o + \angle PSR = 180^o$

$\angle PSR = 75^o$

$SR = TR$ (given)

$\therefore \angle STR = \angle TSR$ (base $\angle$s, isos. $\triangle$)

$= 75^o$

$\angle QRT = \angle STR$ (alt. $\angle$s, $PS // QR$)

$= \underline{\underline{75^o}}$

9. The answer is C.

Let $\angle ABE = 2x$.

$\angle ABE : \angle DCE = 2 : 3$ (given)

$\therefore \angle DCE = 3x$

$\angle BAE + \angle ABE = \angle BEC$ (ext. $\angle$ of $\triangle$)

$\angle BAE + 2x = 90^o$

$\angle BAE = 90^o - 2x$

$\angle BAE = \angle DCE$ (alt. $\angle$s, $AB // DC$)

$90^o - 2x = 3x$

$90^o = 5x$

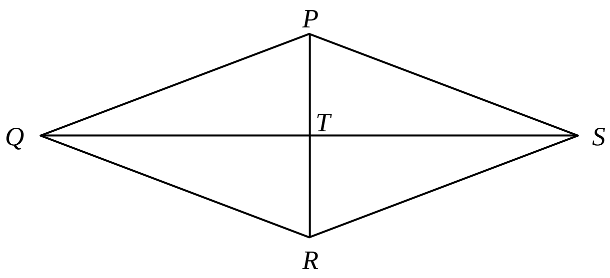
$x = 18^o$

$\angle ABE = 2\,(18^o)$

$= \underline{\underline{36^o}}$

10. The answer is A.

Suppose the diagonals PR and QS meet at T.



$PT = RT = \frac{1}{2}PR$ (property of rhombus)

$= 5$ cm

$QT = ST = \frac{1}{2}QS$ (property of rhombus)

$= 12$ cm

$\angle PTS = 90^\circ$ (property of rhombus)

In $\triangle PTS$,

$PS^2 = PT^2 + ST^2$

$PS = \sqrt{PT^2 + ST^2}$

$= \sqrt{5^2 + 12^2}$ cm

$= 13$ cm

Perimeter of $PQRS$

$= PS \times 4$

$= 13 \times 4$ cm

$= \underline{\underline{52 \text{ cm}}}$

11. The answer is B.

$ST = QT = PT$ (property of rectangle)

$= 2.5a$

$QS = 2ST$

$= (2)(2.5a)$

$= 5a$

$\angle QRS = 90^\circ$ (property of rectangle)

In $\triangle QRS$,

$QS^2 = SR^2 + QR^2$

$(5a)^2 = 7^2 + (5a - 1)^2$

$25a^2 = 49 + 25a^2 - 10a + 1$

$10a = 50$

$a = \underline{\underline{5}}$

12. The answer is D.

The opposite angles of a rhombus are equal.

$\therefore$ $\angle A = \angle C$ and $\angle B = \angle D$.

$\therefore$ The ratio cannot be 2 : 1 : 1 : 2, 1 : 2 : 3 : 4 nor 2 : 1 : 3 : 1.

13. The answer is C.

In $\triangle ABE$ and $\triangle ADF$,

$\angle ABE = \angle ADF$ (property of rhombus)

$AB = AD$

$= AE$ (given)

$\angle AEB = \angle ABE$ (base $\angle$s, isos. $\triangle$)

$= \angle ADF$

$= \angle AFD$ (base $\angle$s, isos. $\triangle$)

$AE = AF$ (given)

$\triangle ABE \cong \triangle ADF$ (AAS)

$\therefore$ $BE = DF$ (corr. sides, $\cong\triangle$s)

$EC = BC - BE$

$= DC - DF$ ($BC = DC$)

$= FC$

i.e. $\angle FEC = \angle EFC$ (base $\angle$s, isos. $\triangle$)

In $\triangle CEF$,

$\angle ECF + \angle FEC + \angle EFC = 180°$ ($\angle$ sum of $\triangle$)

$\angle ECF + 2\angle FEC = 180°$

$$\angle FEC = \frac{180° - \angle ECF}{2}$$

$\angle AEF = 60°$ (property of equilateral $\triangle$)

$\angle ABE + \angle ECF = 180°$ (int. $\angle$s, AB // DC)

$\angle ABE = 180° - \angle ECF$

$\therefore$ $\angle AEB = 180° - \angle ECF$ ($\angle AEB = \angle ABE$)

$\angle AEB + \angle AEF + \angle FEC = 180°$ (adj. $\angle$s on a st. line)

$$180° - \angle ECF + 60° + \frac{180° - \angle ECF}{2} = 180°$$

$$\angle ECF + \frac{\angle ECF}{2} = 150°$$

$$\angle ECF = \underline{\underline{100°}}$$

14. The answer is A.

$\because$ AB // DC and $AB = DC$.

$\therefore$ $ABCD$ is a parallelogram. (2 sides equal and //)

15. The answer is D.

$\because$ $PT = RT$ and $ST = QT$.

$\therefore$ $PQRS$ is a parallelogram. (diags. bisect each other)

16. The answer is A.

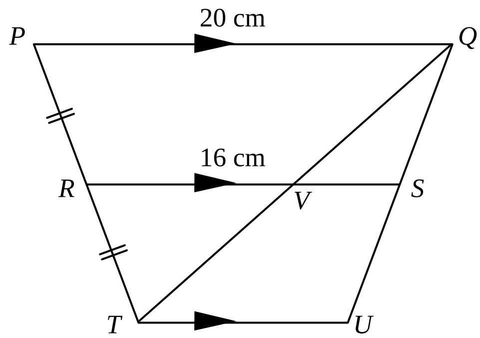
$AM = BM$ and $AN = CN$	(given)
$\therefore\ MN \parallel BC$	(mid-pt. theorem)
$\angle AMN = \angle ABC$	(corr. $\angle$s, $MN \parallel BC$)
$= 70^{\circ}$	
$AM = AN$	(given)
$\therefore \angle ANM = \angle AMN$	(base $\angle$s, isos. $\triangle$)
$x = \underline{\underline{70^{\circ}}}$	

17. The answer is B.

$AC = CE$	(given)
$AB \parallel CD \parallel EF$	(given)
$\therefore\ DF = BD$	(intercept theorem)
x	$= \underline{\underline{2}}$
$AE = EG = 6$	(given)
$BF = FG = 4$	
$AB = 2EF$	(mid-pt. theorem)
$y = \underline{\underline{6}}$	

18. The answer is B.

Join TQ. Suppose TQ and RS meet at V.



$PQ \parallel RS \parallel TU$ and $PR = TR$	(given)
$\therefore\ QS = US$	(intercept theorem)
and $QV = TV$	(intercept theorem)
$\therefore\ VS = \frac{1}{2}TU$	(mid-pt. theorem)
and $RV = \frac{1}{2}PQ$	(mid-pt. theorem)

$$RV + VS = RS$$

$$\frac{1}{2}PQ + \frac{1}{2}TU = RS$$

$$PQ + TU = 2RS$$

$$TU = 2RS - PQ$$

$$= 2(16) - 20 \text{ cm}$$

$$= \underline{\underline{12 \text{ cm}}}$$

19. The answer is A.

$QS = US$ and $PR = UR$ (given)

$\therefore\ RS \,//\, PQ$ and $SR = \frac{1}{2}PQ$ (mid-pt. theorem)

$QS = US$ (given)

$RS \,//\, TU$

$\therefore\ QR = TR$ (intercept theorem)

$QS = US$ and $QR = TR$

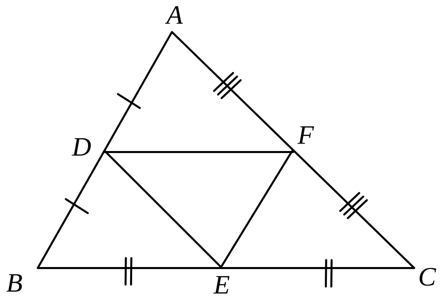
$\therefore\ SR = \frac{1}{2}TU$ (mid-pt. theorem)

$$\therefore\ \frac{1}{2}PQ = \frac{1}{2}TU$$

$$PQ = TU$$

$$x = \underline{\underline{7}}$$

20. The answer is D.



By the mid-point theorem, we have:

$EF \,//\, AB$ and $EF = \frac{1}{2}AB$

$DF \,//\, BC$ and $DF = \frac{1}{2}BC$

$DE \,//\, AC$ and $DE = \frac{1}{2}AC$

$DE \,//\, AC$

$\therefore$ I is true.

$$\frac{EF}{AB} = \frac{FD}{BC} = \frac{ED}{AC} = \frac{1}{2}$$

$\therefore\ \triangle ABC \sim \triangle EFD$ (3 sides proportional)

$\therefore$ III is true.

$$EF = \frac{1}{2}AB$$

$$= AD$$

$$DF = \frac{1}{2}BC$$

$$= EC$$

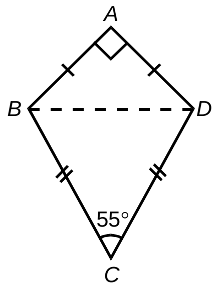
Perimeter of $BEFD$
$= BE + EF + FD + DB$
$= BE + AD + EC + DB$
$= (AD + DB) + (BE + EC)$
$= AB + BC$
$\therefore$ II is true.

MC 2

1. A

$2x + 1 = x + 4$
$x = 3$

2. C



$\because$ $AB = AD$ and $CB = CD$

$\therefore$ $\angle ABD = \angle ADB$ and $\angle CBD = \angle CDB$ (base $\angle$s, isos. Δ)

$\therefore$ $\angle ABC = \angle ADC$

$$\angle ABC + \angle ADC + 90° + 55° = (4-2) \times 180° \quad (\angle \text{ sum of polygon})$$
$$2\angle ABC = 215°$$
$$\angle ABC = 107.5°$$

3. A

Given that AB // DC,

$$\angle BAD + \angle ADC = 180° \quad \text{(prop. of trapezium)}$$
$$2x + 30° + x = 180°$$
$$3x = 150°$$
$$x = 50°$$

4. C

In ΔABD,

$\angle ABD = \angle ADB$ (base $\angle$s, isos. Δ)

$$\begin{aligned} \angle ABD + \angle ADB + \angle BAD &= 180^\circ \quad (\angle \text{ sum of } \Delta) \\ 2\angle ADB + 124^\circ &= 180^\circ \\ 2\angle ADB &= 56^\circ \\ \angle ADB &= 28^\circ \end{aligned}$$

Given that $AD \parallel BC$,

$$\begin{aligned} \angle ADC + \angle DCB &= 180^\circ \quad (\text{prop. of trapezium}) \\ 28^\circ + 90^\circ + x &= 180^\circ \\ x &= 62^\circ \end{aligned}$$

5. D

In ΔABC,

$\angle BAC = \angle BCA$ (base $\angle$s, isos. Δ)

$$\begin{aligned} \angle BAC + \angle BCA + \angle ABC &= 180^\circ \quad (\angle \text{ sum of } \Delta) \\ 2\angle BAC + 4z &= 180^\circ \\ 2\angle BAC &= 180^\circ - 4z \\ \angle BAC &= 90^\circ - 2z \end{aligned}$$

Given that $AB \parallel DC$,

$$\begin{aligned} \angle BAD + \angle ADC &= 180^\circ \quad (\text{prop. of trapezium}) \\ y + (90^\circ - 2z) + 43^\circ &= 180^\circ \\ y &= 47^\circ + 2z \end{aligned}$$

6. C

In ΔCDE,

$$\begin{aligned} \angle DEC + 45^\circ + 90^\circ &= 180^\circ \quad (\angle \text{ sum of } \Delta) \\ \angle DEC &= 45^\circ \end{aligned}$$

$\therefore \quad CD = CE$ (sides opp. eq. $\angle$s)

Given that $AB \parallel DC$,

$$\begin{aligned} \angle ABE + \angle DCE &= 180^\circ \quad (\text{prop. of trapezium}) \\ \angle ABE + 90^\circ &= 180^\circ \\ \angle ABE &= 90^\circ \end{aligned}$$

$$\begin{aligned} \angle AEB + \angle AED + \angle DEC &= 180^\circ \quad (\text{adj. } \angle\text{s on st. line}) \\ \angle AEB + 90^\circ + 45^\circ &= 180^\circ \\ \angle AEB &= 45^\circ \end{aligned}$$

In ΔABE,

$$\begin{aligned} \angle BAE + \angle ABE + \angle AEB &= 180^\circ \quad (\angle \text{ sum of } \Delta) \\ \angle BAE + 90^\circ + 45^\circ &= 180^\circ \\ \angle BAE &= 45^\circ \end{aligned}$$

$\therefore \quad AB = BE$ (sides opp. eq. ∠s)

Area of trapezium $ABCD = \dfrac{(AB+CD)\times BC}{2}$

$= \dfrac{(BE+CE)\times BC}{2}$

$= \dfrac{BC^2}{2}$

$= \dfrac{18^2}{2}\text{ cm}^2$

$= 162\text{cm}^2$

7. D

Given that $AD \parallel BC$,

$\angle ABC + \angle BAD = 180°$ (prop. of trapezium)

$\angle ABC + 70° = 180°$

$\angle ABC = 110°$

In ΔABC,

$\angle BAC + \angle ABC + \angle ACB = 180°$ (∠ sum of Δ)

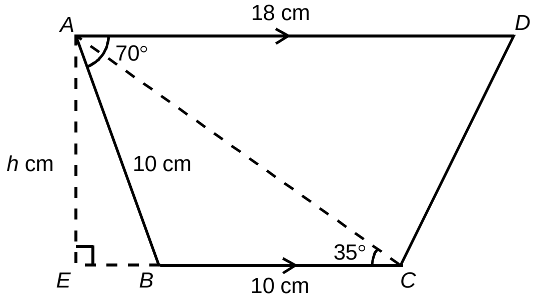
$\angle BAC + 110° + 35° = 180°$

$\angle BAC = 35°$

$\because \quad \angle BAC = \angle ACB = 35°$

$\therefore \quad AB = BC = 10$ cm (sides opp. eq. ∠s)

Let $AE = h$ cm.



Area of $\Delta ABC = 45\text{ cm}^2$

$\dfrac{1}{2}(10)(h) = 45$

$h = 9$

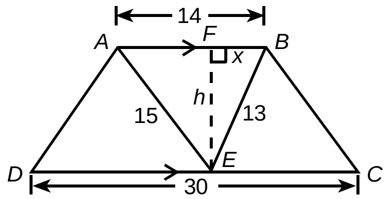
$\therefore \quad AE = 9$ cm

$\therefore \quad$ Area of trapezium $ABCD = [\dfrac{1}{2}\times(10+18)\times 9]\text{cm}^2$

$= 126\text{cm}^2$

8. B

Let h be the height of ΔEAB and $FB = x$.



By the Pythagoras' theorem,

$h^2 + x^2 = 13^2$............................ (1)

$h^2 + (14 - x)^2 = 15^2$............................ (2)

From (1), $h^2 = 13^2 - x^2$..................... (3)

From (2), $h^2 = 15^2 - (14 - x)^2$........... (4)

(3) = (4), $13^2 - x^2 = 15^2 - (14 - x)^2$

$$169 - x^2 = 225 - 196 + 28x - x^2$$
$$28x = 140$$
$$x = 5$$

Substitute $x = 5$ into (1),

$$h^2 + 5^2 = 13^2$$
$$h^2 = 144$$
$$h = 12$$

$\because$ Height of trapezium $ABCD$ = Height of ΔEAB = 12

$\therefore$ Area of trapezium $ABCD = \dfrac{(14+30)\times 12}{2}$

$= 264$

9. C

$AD = BC$ (opp. sides of //gram)

$$x + 12 = 6x - 3$$
$$15 = 5x$$
$$x = 3$$

10. B

$\angle EBC = \angle EDA$ (alt. $\angle$s, AD // BC)

$= 32°$

In ΔEBC,

$\angle EBC + y = 92°$ (ext. $\angle$ of Δ)

$$32° + y = 92°$$
$$y = 60°$$

11. B

In ΔABC and ΔBFE,

$AB = BF = 5$ (given)

$\angle ABC = \angle BFE$ (corr. $\angle$s, BC // FD)

$\angle BAC = \angle FBE$ (corr. $\angle$s, AD // BE)

$\therefore$ $\Delta ABC \cong \Delta BFE$ (A.S.A.)

$\therefore$ $BE = AC = 7$ (corr. sides, $\cong$ Δs)

$BC = FE = 8$ (corr. sides, $\cong$ Δs)

$CD = BE = 7$ (opp. sides of //gram)

$ED = BC = 8$ (opp. sides of //gram)

$\therefore$ Perimeter of parallelogram $BCDE = BC + CD + ED + BE$

$= 8 + 7 + 8 + 7$

$= 30$

12. B

In ΔDEA and ΔBFC,

$DA = BC$ (opp. sides of //gram)

$\angle DEA = \angle BFC = 90°$ (given)

$\angle DAE = \angle BCF$ (alt. $\angle$s, DA // CB)

$\therefore$ $\Delta DEA \cong \Delta BFC$ (A.A.S.)

$\therefore$ $FC = AE$ (corr. sides, $\cong$ Δs)

$= 3$ cm

$\therefore$ $EF = AC - AE - CF$

$= (13 - 3 - 3)$ cm

$= 7$ cm

13. D

$\angle DCB = \angle BAD = 2y$ (opp. $\angle$s of //gram)

$\angle DCF = \angle FCB - \angle DCB$

$= 90° - 2y$

In ΔEFC,

$\angle ECF + x + z = 180°$ ($\angle$ sum of Δ)

$90° - 2y + x + z = 180°$

$x = 90° + 2y - z$

14. A

$2x+15=4x-15$ (definition of square)

$30=2x$

$x=15$

$\therefore$ Perimeter of the square $=(2\times15+15)\times4$

$=180$

15. A

In ΔDEC,

$\angle DEC = \angle DCE$ (base $\angle$s, isos. Δ)

$\angle EDC + \angle DEC + \angle DCE = 180°$ ($\angle$ sum of Δ)

$\angle EDC + 2\angle DEC = 180°$

$\angle EDC = 180° - 2\angle DEC$

$\because$ $\angle ADC = 90°$ (definition of square)

$\therefore$ $\angle EDC < 90°$

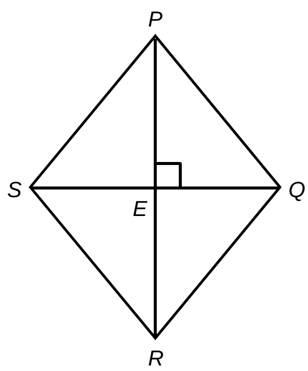
$180° - 2\angle DEC < 90°$

$2\angle DEC > 90°$

$\angle DEC > 45°$

$\therefore$ $\angle DEC$ cannot be $39°$.

16. D



$\angle SER = 90°$ (prop. of rhombus)

In ΔSER,

$SE^2+ER^2=SR^2$ (Pyth. theorem)

$(\frac{SQ}{2})^2+(\frac{PR}{2})^2=SR^2$

$\frac{SQ^2}{4}+\frac{PR^2}{4}=SR^2$

$SR^2=\frac{PR^2+SQ^2}{4}$

17. D

A kite has two pairs of equal adjacent sides. If its sides are not all equal, then the lengths of the two diagonals are not the same.

∴ $AC = BD$ is incorrect.

18. A

Properties of a parallelogram:	(i)	Two pairs of opposite sides are equal.
	(ii)	Two pairs of opposite angles are equal.
	(iii)	The diagonals bisect each other.
Properties of a kite:	(i)	One of the diagonals is the axis of symmetry.
	(ii)	The two diagonals are perpendicular to each other.
Properties of a square:	(i)	It has all the properties of parallelogram.
	(ii)	The diagonals are equal and perpendicular to each other.
	(iii)	The angle between each diagonal and each side is 45°.
Properties of a rhombus:	(i)	It has all the properties of parallelogram.
	(ii)	The diagonals bisect each interior angle.
	(iii)	The diagonals are perpendicular to each other.

∴ "The diagonals are perpendicular to each other" is not the property of a parallelogram.

19. C

∵ A kite has two pairs of equal adjacent sides.

∴ I is correct.

∵ The two diagonals of a kite are perpendicular to each other. However, a quadrilateral with two diagonals perpendicular to each other can be a kite, a square, a rhombus or an irregular quadrilateral.

∴ II is incorrect.

∵ The two diagonals of a square bisect each other. However, a quadrilateral with two diagonals bisect each other can be a parallelogram, a rectangle, a square or a rhombus.

∴ III is incorrect.

20. A

$\because$ $VA = AC = b$ cm, $VB = BD = a$ cm (given)

$\therefore$ $AB = \frac{1}{2}CD$ (mid-point theorem)

$CD = 2AB$

$= (2 \times 8)$ cm

$= 16$ cm

$\because$ $VC = CE = 2b$ cm, $VD = DF = 2a$ cm (given)

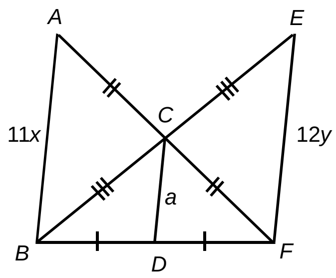
$\therefore$ $CD = \frac{1}{2}EF$ (mid-point theorem)

$EF = 2CD$

$= (2 \times 16)$ cm

$= 32$ cm

21. B



Let a be the length of CD.

$\because$ $AC = CF$, $BD = DF$ (given)

$\therefore$ $CD = \frac{1}{2}AB$ (mid-point theorem)

$a = \frac{1}{2}(11x)$

$x = \frac{2}{11}a$

$\because$ $BC = CE$, $BD = DF$ (given)

$\therefore$ $CD = \frac{1}{2}EF$ (mid-point theorem)

$a = \frac{1}{2}(12y)$

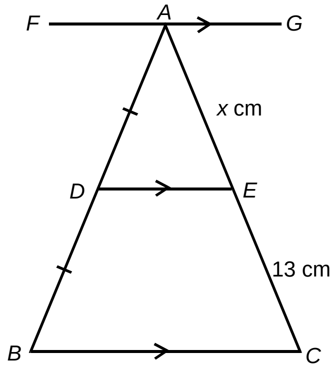
$y = \frac{1}{6}a$

$\therefore$ $x : y = \frac{2}{11}a : \frac{1}{6}a$

$= 12 : 11$

22. B

Draw the straight line FG passing through A and parallel to BC.



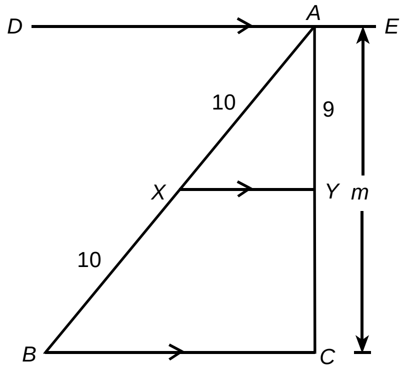
$\because$ $AD = DB$ (given)

$AE = EC$ (intercept theorem)

$\therefore$ $x = 13$

23. C

Draw the straight line DE passing through A and parallel to BC.



$\because$ $AX = XB$ (given)

$AY = YC$ (intercept theorem)

$\therefore$ $YC = 9$

$\therefore$ $m = 9 + 9$

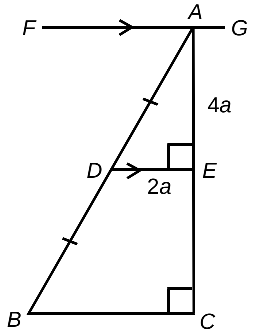
$= 18$

24. D

$\because$ $\angle DEA = \angle BCA = 90°$

$\therefore$ $DE \parallel BC$ (corr. $\angle$s eq.)

Draw the straight line FG passing through A and parallel to BC.



$\because$ $AD = DB$ (given)

$AE = EC$ (intercept theorem)

$EC = 4a + 4a = 8a$

$DE = \frac{1}{2} BC$ (mid-point theorem)

$BC = 2 \times 2a$

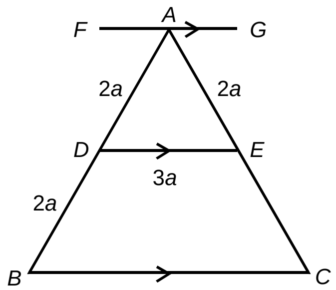
$= 4a$

$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times 4a \times 8a$

$= 16a^2$

25. C

Draw the straight line FG passing through A and parallel to BC.



$\because$ $AD = DB$ (given)

$AE = EC$ (intercept theorem)

$EC = 2a$

$\therefore$ $AC = 2a + 2a = 4a$

$DE = \frac{1}{2} BC$ (mid-point theorem)

$BC = 2 \times 3a$

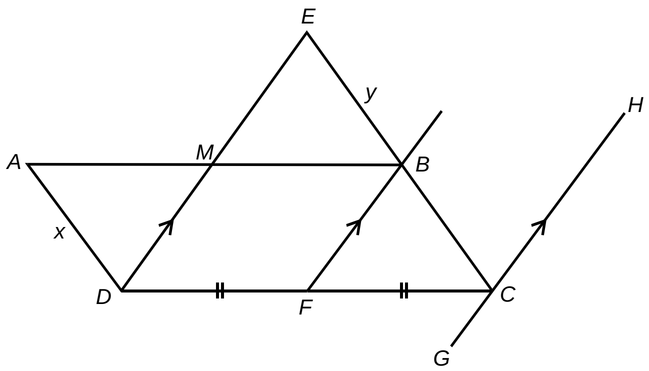
$= 6a$

$\therefore$ Perimeter of $\Delta ABC = 4a + 4a + 6a$

$= 14a$

26. C

Draw the straight line GH passing through C and parallel to DE.



$\because$ $CF = FD$ (given)

$CB = y$ (intercept theorem)

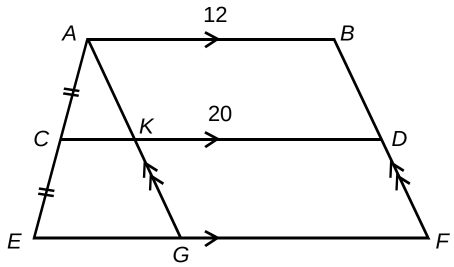
$\because$ $AD = BC$ (opp. sides of //gram)

$x = y$

$\therefore$ $\frac{x}{y} = 1$

27. B

Draw the straight line AG parallel to BF, such that it intersects CD at K.



$AB = KD = GF = 12$ (opp. sides of //gram)

$CK = CD - KD$

$= 20 - 12$

$= 8$

$\because$ $AC = CE$ (given)

$\therefore$ $AK = KG$ (intercept theorem)

$\therefore$ $CK = \frac{1}{2}EG$ (mid-point theorem)

$EG = 2 \times 8$

$= 16$

$\therefore$ $EF = EG + GF$

$= 16 + 12$

$= 28$